Monopoly

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June 14, 2023

Introduction

- Markets have varying levels of competition
- One extreme: perfect competition
  - Many identical firms
  - Price takers
  - Free entry
  - Zero economic profits in the long run
- The other extreme: pure monopoly
  - Only one firm
  - Effectively sets their own price
- We'll talk now about monopolistic markets

Monopoly vs Perfect Competition

- In a perfectly competitive setting, we said firms were *price takers*, meaning their decisions had no impact on the market price.
- Total revenue in this case was given by:

$$TR(p, q) = pq$$

- A monopolist on the other hand does influence market prices. Their total revenue is given by:

$$TR(p, q) = p(q)q$$

- The difference? Market prices explicitly depend on the monopolist's production decision

Profit Maximization

- A monopolist's profits are given by:

$$\pi(q) = p(q)q - c(q)$$

- Optimal quantity q^* satisfies:

$$MR(p, q) = MC(q)$$
$$p'(q)q + p(q) = c'(q)$$

- We have the same optimality condition as before $(MR = MC)$
- However, marginal revenue is different in a monopolistic setting
  - Perfect competition: $MR(p, q) = p$
  - Monopoly: $MR(p, q) = p'(q)q + p(q)$

Marginal Revenue

$$MR(p, q) = p(q) + p'(q)q$$

- For a monopolist, how does increasing q by 1 impact total revenue?
- There are two effects:
  1. Produce 1 more q, get one more p (first term)
  2. However, increasing q changes p (second term)
- Monopolist must consider both effects when making production decisions
- Let's work through an example

Profit Maximization (Example)

- Consider a monopolist with the following cost function:

$$c(q) = 5 + 2q^2$$

- Market demand is given by:

$$Q^d(p) = 60 - p$$

- What is the profit maximizing level of output q^*?
- First, plug inverse demand into the profit equation:

$$\begin{aligned}\pi(q) &= p(q)q - 5 - 2q^2 \\ &= (60 - q)q - 5 - 2q^2\end{aligned}$$

Profit Maximization (Example)

$$\pi(q) = (60 - q)q - 5 - 2q^2$$
$$= 60q - q^2 - 5 - 2q^2$$

- Optimal quantity satisfies:

$$60 - 2q = 4q$$
$$q^* = 10$$

- It is optimal for the monopolist to produce $q^* = 10$ units

Profit Maximization (Example)

$$q^* = 10$$

- Remember that a monopolist effectively sets their own price
- How much do they charge per unit in this example?
- We can obtain price by plugging q^* into the inverse demand function:

$$p^* = 60 - q^* = 60 - 10 = 50$$

- In the end, we find that the monopolist produces $q^* = 10$ units and charges $p^* = 50$ per unit

Marginal Revenue & Elasticity

- It is useful to rewrite marginal revenue in terms of elasticities
- The elasticity of demand is given here by:

$$\epsilon = \frac{\partial q}{\partial p}\frac{p}{q}$$

- Rewriting marginal revenue, we see that:

$$\begin{aligned} MR(p,q) &= p + \frac{\partial p}{\partial q} q \\ &= p + p\frac{\partial p}{\partial q}\frac{q}{p} \\ &= p\left(1 + \frac{1}{\epsilon}\right) \end{aligned}$$

Marginal Revenue & Elasticity

$$MR(p,q) = p\left(1 + \frac{1}{\epsilon}\right)$$

- A monopolist's marginal revenue depends on how elastic consumer demand is
- We're always assuming the law of demand holds, so $\epsilon < 0$
- Perfectly elastic demand ($\epsilon = \infty$): $p = MR$
  - This is what we see in perfect competition
- Elastic demand ($\epsilon < -1$): $MR > 0$
- Unit elastic demand ($\epsilon = -1$): $MR = 0$
- Inelastic demand ($\epsilon > -1$): $MR < 0$

Lerner Index

- How much market power does a firm have?
- One way to quantify market power is by using the *Lerner Index* (or price markup)
- Recall that for a monopolist:

$$MC(q) = p\left(1 + \frac{1}{\epsilon}\right)$$

- We can use this to derive the Lerner Index L:

$$L = \frac{p - MC}{p} = -\frac{1}{\epsilon}$$

Lerner Index

$$L = \frac{p - MC}{p} = -\frac{1}{\epsilon}$$

- The Lerner Index simply measures how high price is relative to marginal cost
- For a profit-maximizing firm, L is a number between 0 and 1
  - Always produces where $|\epsilon| \geq 1$
- If $p = MC$ (perfect competition), then $L = 0$ and the firm has no market power
- L gets closer to 1 as the firm has more market power
  - Charges a higher markup

Lerner Index

$$L = -\frac{1}{\epsilon}$$

- Notice that more inelastic demand yields higher market power
- In other words, as ϵ approaches -1 (demand becomes relatively less elastic), L increases
- For example:
  - Pharmaceutical industry: very inelastic demand, firms can charge extremely high markups

Taking Inventory

- Like any firm, monopolists produce until $MR = MC$
- The marginal revenue curve for a monopolist is given by:

$$MR(q) = p'(q)q + p(q)$$

- This can be expressed in terms of consumers' elasticity of demand:

$$MR(q) = p\left(1 + \frac{1}{\epsilon}\right)$$

- We can use this expression to define the *Lerner Index*, which measures the monopolist's market power
- Let's take a moment to compare and contrast a perfectly competitive market with a monopolistic market

Monopoly vs Perfect Competition

- Recall the setup for a perfectly competitive market
- N identical firms, each has the same cost function $c(q)$
- Individual firm profits are given by:

$$\pi(q) = pq - c(q)$$

- Firms are *price takers*, meaning p is taken as a fixed constant

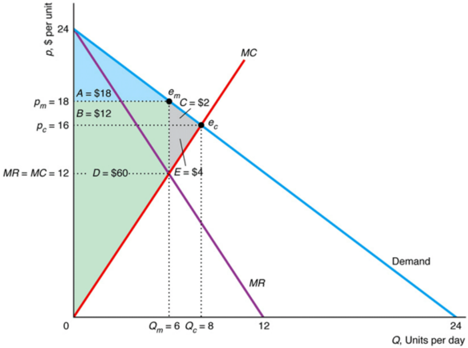
Monopoly vs Perfect Competition

- Marginal revenue in a perfectly competitive market is equal to inverse demand:
$$MR(q) = p$$
- Monopolists on the other hand control their own price. Their profits are given by:
$$\pi(q) = p(q)q - c(q)$$
- The market price $p(q)$ explicitly depends on the monopolist's production decision

Monopoly vs Perfect Competition

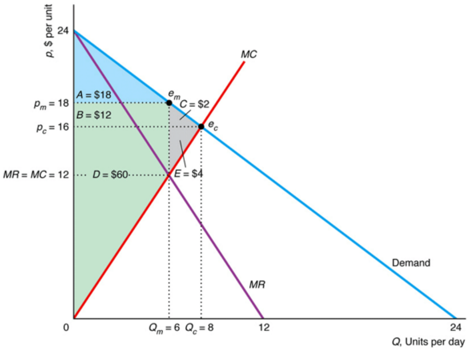
- For a monopolist, marginal revenue is not just equal to inverse demand:
$$MR(q) = p'(q)q + p(q)$$
- It can be shown that a monopolist's marginal revenue curve always has twice the slope as the inverse demand curve:
$$MR'(q) = 2p'(q)$$
- Always true when inverse demand is linear
- Let's look at this graphically

Market Power and Welfare

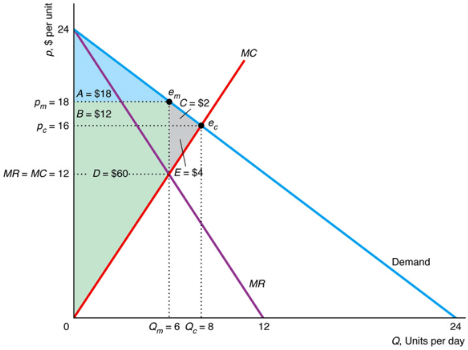


- Marginal revenue and inverse demand are not equal for a monopolist
- Monopolists produce until $MC = MR \neq p$

Market Power and Welfare

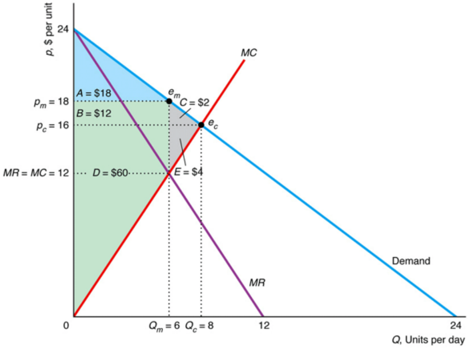


- Consumer surplus (CS) can be computed as the area of triangle "A" above
- Producer surplus (PS) can be computed as the area of the green trapezoid



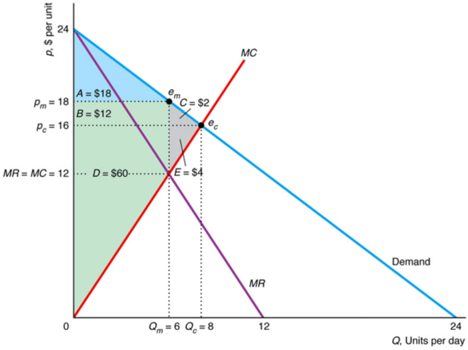
- Total surplus (TS) is the sum of CS and PS
- The surplus (or welfare) maximizing level of quantity equates MC and inverse demand

Market Power and Welfare

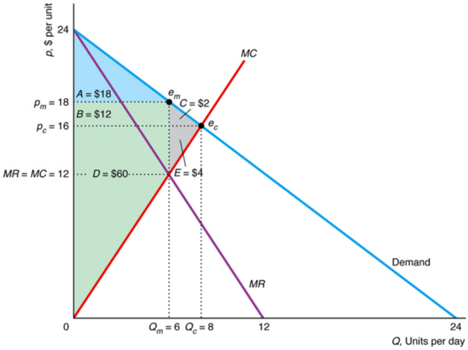


- In a perfectly competitive market, equilibrium quantity is equal to the surplus-maximizing level of quantity
- In a monopolistic market, the quantity supplied by the firm is strictly less than the surplus-maximizing quantity

Market Power and Welfare



- Monopolists intentionally generate shortages, which allows them to charge higher prices
- This shortage destroys welfare (i.e. creates "deadweight loss")



- The DWL, the lost surplus, is represented by the grey triangle
- The area of the grey triangle, "C"+"E", gives the total amount of DWL

Market Power and Welfare

- In summary: monopolists produce less than the socially optimal level
- Why? Because it allows them to charge higher prices.
- Relative to the PC case, monopolists receive additional surplus at the expense of consumers
- However, they generate deadweight loss, so total surplus (welfare) under a monopoly is always less than that of a PC market
- Big picture: competition is good for consumers, but bad for firm profits

Example

- Let's conclude this section with an example
- Consider a monopolist with the following cost function:

$$c(q) = 10 + q^2$$

- Market demand is given by:

$$Q^d = 40 - p$$

- First, what is the equilibrium price p^* and quantity q^*?

Example

- The monopolist's profits are given by:

$$\pi(q) = p(q)q - 10 - q^2$$

- Use market depend to substitute in $p(q)$:

$$\begin{aligned}\pi(q) &= (40 - q)q - 10 - q^2 \\ &= 40q - q^2 - 10 - q^2\end{aligned}$$

- Maximizing firm profits gives:

$$\begin{aligned}40 - 2q - 2q &= 0 \\ q^* &= 10\end{aligned}$$

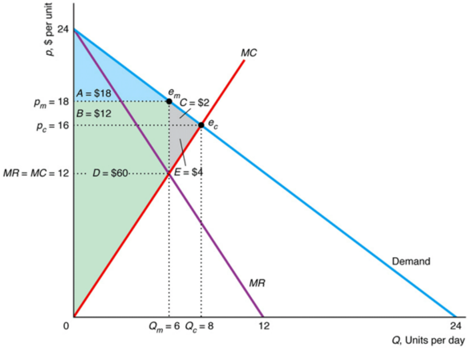
Example

- The profit-maximizing quantity is $q^* = 10$. How about the price?
- Plug q^* into the market demand curve:

$$10 = 40 - p$$
$$p^* = 30$$

- The equilibrium of this market is summarized by $(q^*, p^*) = (10, 30)$
- Next, we can compute consumer and producer surplus

Example (Consumer Surplus)



- Consumer surplus is given by the area of triangle A
- Let's go over how to compute this

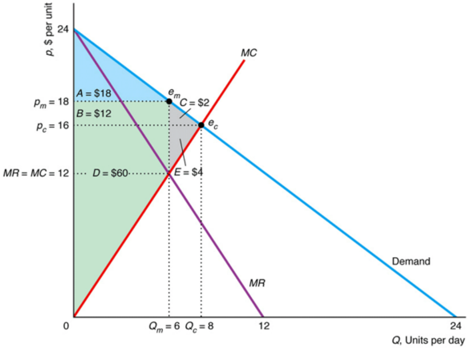
Example (Consumer Surplus)

- A has width equal to quantity: $q^* = 10$
- A has height equal to: $p(0) - p(q^*)$ where p is the inverse market demand curve
  - Simply the intercept of inverse demand minus equilibrium price
- Area of a triangle is length $\times$ width $\times$ $\frac{1}{2}$
- In this example:

$$
\begin{aligned}
CS &= \frac{1}{2} q^* (p(0) - p(q^*)) \\
&= \frac{1}{2}(10)(40 - 30) \\
&= 50
\end{aligned}
$$

- Next, we can compute producer surplus

Example (Producer Surplus)



- Producer surplus is given by the area of the green trapezoid
- Consists of a rectangle and a triangle
- Let's talk about computing this one

Example (Producer Surplus)

- Formula for computing producer surplus:

$$PS = q^* p^* - \frac{1}{2} q^* MC(q^*)$$

- Quantity equals: $q^* = 10$
- Price equals: $p^* = 30$
- Marginal cost (at q^*) equals: $MC(q^*) = 20$
- Producer surplus is then:

$$(10)(30) - \frac{1}{2}(10)(20) = 300 - 100 = 200$$

Example (Producer Surplus)

- In summary, $PS = 200$ while $CS = 50$
- In a monopolistic market, the firm always receives a greater share of the total surplus than consumers
- Monopolies induce shortages enabling them to fix prices about the socially-optimal level
- Doing so increase producer surplus, but decreases consumer and total surplus