# Perfect Competition

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# Introduction

- We've talked a lot about optimal firm decision making
    - Firms can optimize both costs and quantity
- However, firms don't make decisions in a void
- They participate in an overarching *market*, the structure of which will have implications for firms' optimal production decisions
- We'll focus on three forms of markets:
    1. Perfectly competitive markets
    2. Monopolistic markets
    3. Oligopolies
- We'll start with the case of perfect competition

# Perfect Competition

- Characteristics of a perfectly competitive market:
  - Large number of firms (small barriers to entry)
  - Undifferentiated (identical) products
  - Large number of buyers
  - Perfect information for firms & consumers
  - Zero economic profits
  - Firms are price takers
- Firms in a perfectly competitive market take prices as given
  - i.e. their production decision has no impact on the market price
- Total revenue is then given simply by:

$$TR(p, q) = pq$$

# Perfect Competition

$$TR(p, q) = pq$$

- In general, total revenue is given by $q$ times the *inverse demand curve* $p(q)$
- In a perfectly competitive setting, inverse demand is just a constant
- Firms in a perfectly competitive market thus face a horizontal demand curve
  - Consumer demand is "perfectly" elastic

# Perfect Competition

- Firm profits in a perfectly competitive market are given by:

$$\pi(q) = pq - c(q)$$

- The optimal production decision satisfies:

$$MC(q) = p$$

- Firms produce until marginal cost equals price
- Note that $p = MC$ is identical to $MR = MC$ given perfect competition since $MR = p$

# Perfect Competition (Example)

$$\pi(q) = pq - 2q^2$$

- As a simple example, suppose that a firm's profits are given above
- Again, to derive optimal quantity, we differentiate with respect to $q$ and set equal to zero:

$$p - 4q = 0$$
$$q^* = \frac{p}{4}$$

- Optimal quantity is $q^* = \frac{p}{4}$. How much does this generate in profits?

# Perfect Competition (Example)

- Plugging $q^*$ into the profit equation yields:

$$\pi(q^*) = pq^* - 2(q^*)^2$$
$$\pi(q^*) = p\left(\frac{p}{4}\right) - 2\left(\frac{p}{4}\right)^2$$
$$= \frac{p^2}{4} - \frac{p^2}{8} = \frac{p^2}{8}$$

- The best the firm can do is produce $\frac{p}{4}$ units of $q$, which yields $\frac{p^2}{8}$ in profits
- Firm attains some positive profits in this example

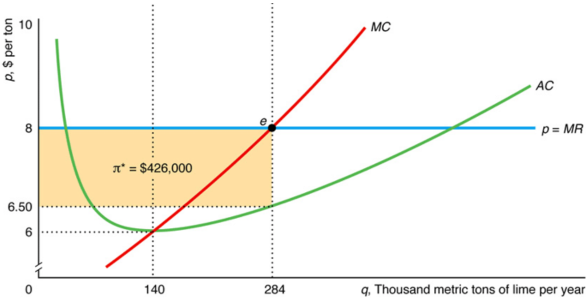
# Perfect Competition

- In order to visualize firm profits graphically, it is useful to observe:

$$\begin{aligned}\pi(q) &= pq - c(q) \\ &= q\left(p - \frac{c(q)}{q}\right) \\ &= q(p - ATC(q))\end{aligned}$$

- Total profits can always be computed as quantity $q$ times the difference in $p$ and $ATC(q)$
- Let's take a look at this graphically

# Firm Profits Visualized




- Profits can be obtained as the area of the rectangle above
- Rectangle has length equal to $(p - ATC(q))$, and width equal to $q$

# Shutdown Decision

- As long as $p > ATC(q)$, firms make positive profits
- However, it is possible that firms incur a loss, yet still find it optimal to stay open in the short run
- Recall the shutdown condition:

$$TR(p, q) < VC(q)$$

- Firms shut down if and only if total revenue is less than variable costs

# Shutdown Decision

- In a perfect competition setting, the shutdown condition is:

$$pq < VC(q)$$

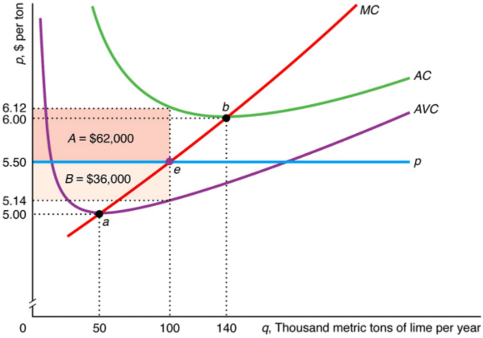
- Dividing by $q$, we see the condition above is equivalent to:

$$p < AVC(q)$$

- In a perfectly competitive market, firms shut down if and only if the price falls short of average variable costs
- Firms incur a loss (but stay open) if:

$$ATC(q) > p > AVC(q)$$

# Firm Losses Visualized




- If $ATC(q) > p$, firms will incur a loss
- Their total loss can be computed as the area of rectangle "A"

# Taking Inventory

- In summary:
  - Positive profits if: $p > ATC(q)$
  - Negative profits if: $ATC(q) > p > AVC(q)$
  - Shut down if: $AVC(q) > p$
- We can use these insights to think carefully about firms' quantity decisions (i.e. their supply curves)

# Supply Curves

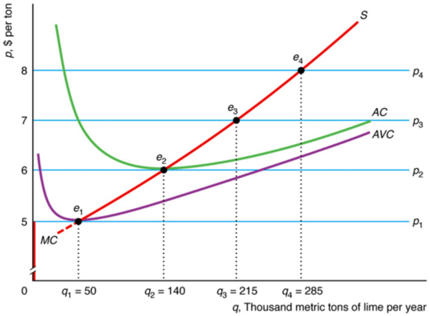
- A firm's supply curve $q_i^s(p)$ describes their optimal quantity as a function of price $p$
- Due to the shutdown condition, firms choose $q_i^s(p) = 0$ if $p < AVC(q)$
- Supply curves are generally given by:

$$q_i^s(p) = \begin{cases} q^* & \text{if} \quad p \geq AVC(q) \\ 0 & \text{if} \quad p < AVC(q) \end{cases}$$

- $q^*$ is the quantity satisfying:

$$p = MC(q^*)$$

# Supply Curves




- In a PC setting, the supply curve is the portion of the $MC(q)$ curve above $AVC(q)$

# Market Supply

- Consider a market with $N$ firms
- Each firm has the supply curve:

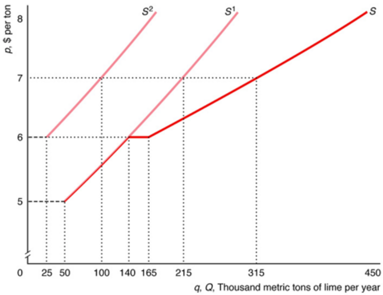
$$q_i^s(p) \quad \text{for} \quad i = 1, \ldots, N$$

- Recall the definition of "aggregate" or market supply:

$$Q^s(p) = \sum_{i=1}^{N} q_i^s(p) = q_1(p) + \ldots + q_N(p)$$

- The market supply curve is simply the sum of the individual supply curves

# Market Supply




- Aggregate supply is the sum across each of the individual supply curves

# Market Demand

- Suppose there are $C$ consumers in this market
- Each consumer has the demand function:

$$q_i^d(p) \quad \text{for} \quad i = 1, \ldots, C$$

- Similarly, market demand is given by:

$$Q^d(p) = \sum_{i=1}^{C} q_i^d(p) = q_1^d(p) + \ldots + q_C^d(p)$$

- Market demand, or "aggregate" demand, is simply the sum of the individual demands

# Short-Run Market Equilibrium

- A market is in *equilibrium* if market supply equals market demand:

$$Q^s(p) = Q^d(p)$$

- The condition above is referred to as the *market clearing* condition
- The price satisfying market clearing is the market price $p^*$
- Additionally, all firms maximize profits:

$$p = MC(q_i)$$

- Given a price $p^*$, firms produce $Q^*$
- An equilibrium is summarized by the pair $(Q^*, p^*)$
- Let's work through an example

# Short-Run Market Equilibrium (Example)

- Consider a market with $N = 300$ identical firms
- Each firm has the cost function:

$$c(q_i) = 25 + 150q_i^2$$

- Suppose we've derived the following market demand:

$$Q^d(p) = 60 - p$$

- What is the equilibrium of this market?

# Short-Run Market Equilibrium (Example)

- Step one: derive the market supply curve
- Firm profits are given by:

$$\pi(q_i) = pq_i - 25 - 150q_i^2$$

- Individual supply is then given by:

$$q_i^s(p) = \frac{p}{300}$$

- To obtain aggregate supply, just multiply individual supplies by $N = 300$:

$$Q^s(p) = \sum_{i=1}^{300} q_i^s(p) = \sum_{i=1}^{300} \frac{p}{300} = p$$

# Short-Run Market Equilibrium (Example)

- We now have both market supply and market demand. Setting them equal:

$$Q^d(p) = Q^s(p)$$
$$60 - p = p$$
$$p^* = 30$$

- The equilibrium price is thus 30. To obtain equilibrium quantity, plug this into either the market supply or demand curve.
    - Can use either since they are equal in equilibrium

- What we obtain in the end is $(p^*, q^*) = (30, 30)$

# Deriving Short-Run Market Equilibria (Summary)

- To derive the market equilibrium $(p^*, q^*)$:
  1. Derive market supply
  2. Use market clearing to derive $p^*$
  3. Plug $p^*$ into market supply or demand to obtain $q^*$
- Let's go through a slightly more general example

# Short-Run Market Equilibrium

- The market has $N$ identical firms
- Firms have a convex cost function and profits given by:

$$\pi(q) = pq - \frac{1}{2}q^2$$

- Market demand is linear and decreasing in $p$:

$$Q^d(p) = a - bp$$

- $b$ measures consumers' sensitivity to price

# Short-Run Market Equilibrium

- Aggregate supply is given by: $Q^s(p) = Np$
- The market price is given by:

$$p^* = \frac{a}{b+N}$$

- Firm profits are given by:

$$\pi(q^*) = \frac{a^2}{2(b+N)^2} > 0$$

- Firms make a positive profit here

# Long-Run Equilibrium

$$\pi(q^*) = \frac{a^2}{2(b+N)^2}$$

- If this market is profitable, other firms will be drawn to the market, increasing $N$
- Notice that as $N$ increases, firm profits are driven to zero
- More firms $\rightarrow$ more competition $\rightarrow$ lower prices
- **Takeaway:** In the long run, opportunities for profit draw firms into the market. As the number of firms increases, profits converge to zero.

# Long-Run Equilibrium

- A *long-run* market equilibrium is defined by the following conditions
- Firms maximize profits:

$$MC(q_i^*) = p^*$$

- Markets clear:

$$Q^s(p^*) = Q^d(p^*)$$

- Additionally, firms make zero profit:

$$p^* = ATC(q_i^*)$$

# Zero Economic Profits

- A quick digression: what do we mean by zero profits?
- It is important to distinguish between:
  - Accounting profit
  - Economic profit
- Accounting profit: total revenue minus accounting cost
  - Actual dollar value of profits
- Economic profit: total revenue minus accounting cost & opportunity cost
  - Dollar value of profits minus opportunity cost
- Economic profit measures how much better participating in the market is relative to the next best alternative

# Zero Economic Profits

- When we say "zero profits," what we really mean is zero *economic profits*
- Economic profits are defined using the market prices of all production inputs
- Market prices reflect the opportunity cost of those factors of production
  - Firm could have used their $L$ and $K$ for something else
- Where does opportunity cost pop up in the cost equation? It essentially pops up in the input prices $w$ and $r$

# Long-Run Equilibrium

- Back to long-run equilibrium. It is characterized by the three conditions:

$$\begin{aligned} MC(q_i^*) &= p^* \\ Q^s(p^*) &= Q^d(p^*) \\ p^* &= ATC(q_i^*) \end{aligned}$$

- Same as a short-run equilibrium, but we add the additional condition ensuring firms make zero profits
- Let's go through an example of deriving a long-run market equilibrium

# Long-Run Equilibrium (Example)

- $N$ identical firms, each has cost function:

$$c(q) = 40q - q^2 + .01q^3$$

- Market demand is given by:

$$Q^d(p) = 25000 - 1000p$$

- Three things we need to determine:
  1. Long-run equilibrium quantity $Q^*$
  2. Long-run equilibrium price $p^*$
  3. How many firms enter the market? ($N^*$)

# Long-Run Equilibrium (Example)

$$\pi(q_i) = pq_i - 40q + q_i^2 - .01q_i^3$$

- Starting with firms' profit maximization condition:

$$p = MC(q_i)$$
$$p = 40 - 2q_i + .03q_i^2$$

- Next for the zero-profit condition:

$$p = ATC(q_i)$$
$$p = 40 - q_i + .01q_i^2$$

- Since both $MC(q_i)$ and $ATC(q_i)$ equal $p$, we can set them equal

# Long-Run Equilibrium (Example)

- Equating $MC(q_i)$ and $ATC(q_i)$, we can derive individual supply:

$$40 - 2q_i + .03q_i^2 = 40 - q_i + .01q_i^2$$
$$.02q_i^2 = q_i$$
$$q^* = 50$$

- We can plug this into the zero-profit condition to obtain the equilibrium price:

$$p = 40 - (50) + .01(50)^2$$
$$p^* = 15$$

- Lastly, we need to determine how many firms enter the market ($N^*$). For this, we can use the market clearing condition.

# Long-Run Equilibrium (Example)

- Market clearing requires that:

$$\begin{aligned} Q^d(p) &= Q^s(p) \\ 25000 - 1000p &= Nq_i^* \\ 25000 - 1000(15) &= N(50) \\ N^* &= 200 \end{aligned}$$

- In the long run, $N^* = 200$ firms participate in this market

# Deriving Long-run Market Equilibria (Summary)

- In summary, to derive a long-run market equilibrium, we need three things:
  - Equilibrium quantity $Q^s = q^* \times N^*$
  - Equilibrium price $p^*$
  - Equilibrium number of firms $N^*$
- Obtain $q^*$ by setting $MC(q_i) = ATC(q_i)$
- Obtain $p^*$ by plugging $q^*$ into either the profit-maximizing or zero-profit condition
- Obtain $N^*$ using the market-clearing condition

# Market Selection

- We mentioned that as more and more firms enter the market, prices are driven down
- Remember, if $p < AVC(q_i)$, firm $i$ prefers shutting down over continuing to participate in the market
- As prices fall, some firms will inevitable shut down
- In particular, high AVC firms will be driven out of the market by low AVC firms
- This is phenomenon is what's at the heart of *market selection*

# Market Selection

- We can think about the evolution of markets using biological evolution as an analogy
- Natural selection: species with favorable traits survive while species with unfavorable traits die out
- Market selection: firms with favorable qualities (low costs) survive while firms with unfavorable qualities (high costs) eventually die out
- The market favors firms who can produce efficiently (i.e. produce at relatively low cost)

# Market Selection (Summary)

- Profit opportunities draw firms into the market
- As the number of firms in the market increases, competition increases, driving prices (and profits) down
- As prices fall, high-cost firms are gradually "priced out" of the market
- The more efficient firms, those who can produce at relatively low cost, survive