Income & Substitution Effects

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Introduction

- In the previous section, we took a close look at individuals' demand functions
- The demand function $x(p_x, p_y, I)$ gives the optimal quantity of x to be consumed given prices (p_x, p_y) and income (I)
- By differentiating x's demand function with respect to its price p_x, we learn how consumption of x changes with its price
- This consumption change can be decomposed into "smaller" components, which will be the focus of this chapter

$$x = \frac{I}{2p_x}$$

- Suppose that we've derived the demand function above
- Since $\frac{\partial x}{\partial p_x} < 0$, the LOD is satisfied
- p_x decreases $\rightarrow$ buy more x
- There is more we can say about the effects of a price change
- In particular, we can decompose the change in x into two pieces:
  1. Income effect
  2. Substitution effect

Income & Substitution Effects

- Suppose that p_x decreases
- Two things happen when p_x changes
- First, x becomes cheaper relative to y
  - Pushes $\uparrow$ demand for x, pushes $\downarrow$ demand for y
  - *Substitution effect*
- Second, prices fall, and I have more purchasing power
  - Pushes $\uparrow$ demand for normal goods, pushes $\downarrow$ demand inferior goods
  - *Income effect*
- Adding up both effects gives us the *total effect*
  - i.e. the total observed change in demand

Slutsky Equation

$$TE = IE + SE$$

- The above identity is known as the **Slutsky Equation**
- When p_x changes, the "total effect" (TE) is just the total observed change in x
- TE is the sum of the income effect (IE) and substitution effect (SE)
- How much of the consumption change is due to:
  - Relative change in prices? (SE)
  - Increase in purchasing power? (IE)

Example: Minimum wage increase

- Income & substitution effects are useful when thinking about the effects of wage changes
- What if the minimum wage increased from \$7.25 to \$15?
  - Would people increase their desired number of work hours?
  - Probably
- What is the minimum wage increased from \$7.25 to $\$10,000$?
  - Would people choose to spend all of their time working?
  - Probably not
- Let's organize our thoughts about this

Example: Minimum wage increase

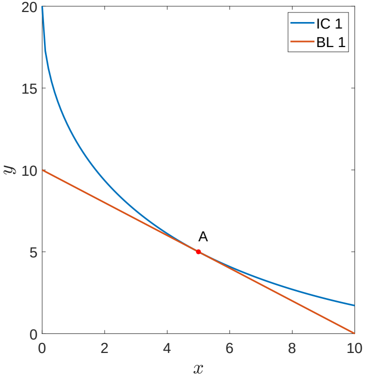
- Suppose that minimum wage increases
- On the one hand, wages are higher, people want to work more
  - Upward force on labor hours, downward force on leisure hours
  - Substitution effect
- On the other hand, wages are higher, people need not work as much to make a living
  - Upward force on leisure hours, downward force on work hours
  - Income effect
- Does a minimum wage hike increase or decrease labor supply? This depends on whether the IE or SE dominates

Computing the IE and SE

$$TE = IE + SE$$

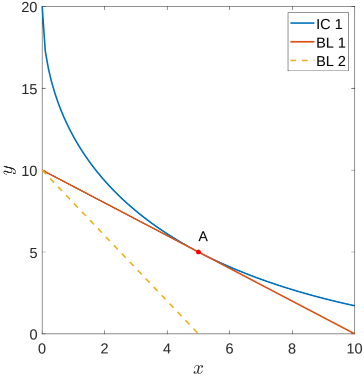
- Back to the Slutsky equation
- The total effect for x is given simply by the own-price derivative $\frac{\partial x}{\partial p_x}$
- How do we separate this into the SE and IE?
- Let's look at it graphically, then work through an example

IE and SE: Graphical Representation



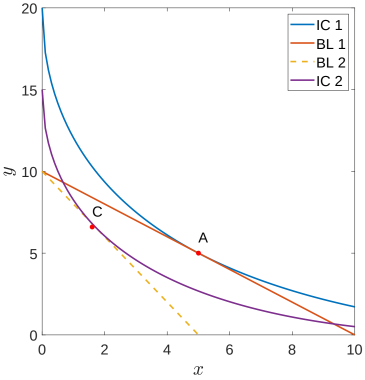
- Suppose we're given some utility function $u(x, y)$
- Given some prices (p_x, p_y), and income I, the optimal bundle is "A"

IE and SE: Graphical Representation



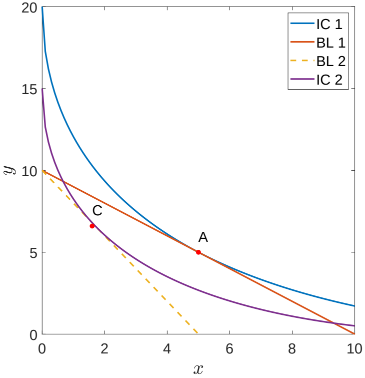
- Now suppose p_x increases
- These induces an inward rotation of the budget line
  - Call it “BL 2”

IE and SE: Graphical Representation



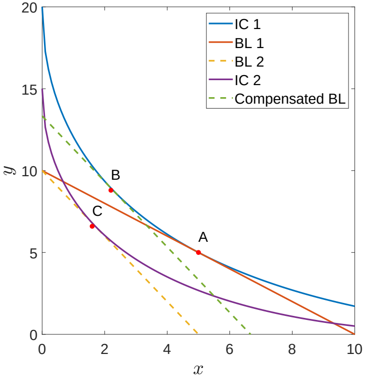
- Under this new budget constraint, we choose a new consumption bundle
- Call the new optimal bundle "C"

IE and SE: Graphical Representation



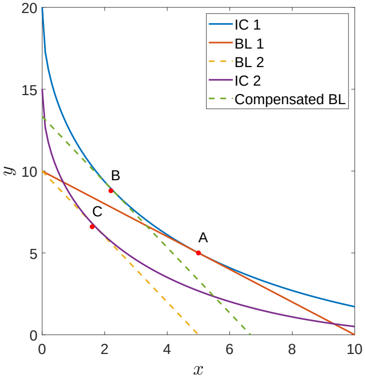
- The change in x going from point A to point C is x's TE
  - Similar for y
- To decompose the TE, we need an “intermediate” point

IE and SE: Graphical Representation



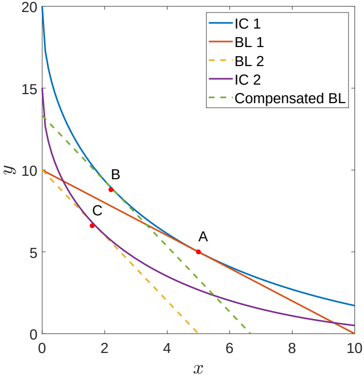
- Consider the “compensated” budget line:
  - Same slope as BL 2
  - Tangent to IC 1

IE and SE: Graphical Representation



- Going from point A to point B gives the SE
- Going from point B to point C gives the IE

IE and SE: Graphical Representation



- We've split the total effect into a:
  - **Pivot** along the same IC (SE)
  - **Shift** to another IC (IE)

Computing the IE and SE

$$u(x, y) = x^{1/2}y^{1/2}$$

- How do we compute IEs and SEs numerically?
- Suppose we're given the utility function above, and consider the following scenario:
  - Initially, $p_x = 8$, $p_y = 2$, and $I = 400$
  - Then, price of y increases to $p_y = 8$
- We need to compute three bundles:
  - Point A - optimal bundle with original prices
  - Point B - tangency point between IC 1 and compensated BL
  - Point C - optimal bundle with new prices

Computing the IE and SE

- We'll first compute the optimal bundle when $p_y = 2$ (Point A)
- Let MRT_1 and MRT_2 respectively be the original and new price ratios
- Setting $MRS = MRT_1$:

$$\frac{y}{x} = 4$$
$$y = 4x$$

- Plugging this into the budget line:

$$400 = 8x + 2y$$
$$400 = 8x + 2(4x)$$
$$x^* = 25$$

- The first optimal bundle is thus $(x^*, y^*) = (25, 100)$

Computing the IE and SE

- Next, we'll compute the optimal bundle when $p_y = 8$ (Point C)
- Setting $MRS = MRT_2$:

$$\frac{y}{x} = 1$$
$$y = x$$

- Plugging this into the budget line:

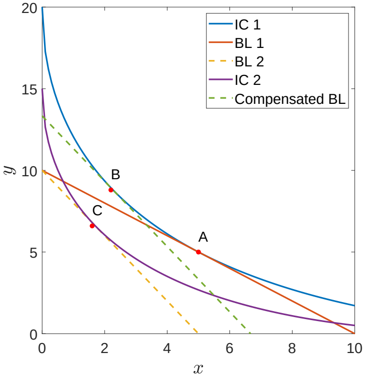
$$400 = 8x + 8y$$
$$400 = 8x + 8(x)$$
$$x^* = 25$$

- The new optimal bundle is thus $(x^*, y^*) = (25, 25)$

Computing the IE and SE

- So far, we've computed two bundles
- When $p_y = 2$, $(x^*, y^*) = (25, 100)$
- When $p_y = 8$, $(x^*, y^*) = (25, 25)$
- When p_y increased to 8:
  - The TE on x is 0
  - The TE on y is -75
- To decompose this into the IE and SE, we'll need to compute the hypothetical "intermediate" bundle (Point B)

Computing the IE and SE



- At point B, we get the same utility as Point A
- But we face the same MRT as Point C

Computing the IE and SE

- To compute Point B, we use the two conditions:

$$MRS = MRT_2$$
$$u(x_B, y_B) = u(x_A, y_A)$$

- $MRS = MRT$ condition using the *new* prices
- Make sure we get the same utility as with Point B
- Note here the utility we got with Point A:

$$u(x, y) = u(25, 100)$$
$$u(x, y) = 25^{1/2}100^{1/2}$$
$$u(x, y) = 50$$

Computing the IE and SE

- Then, in this particular example, our two conditions are:

$$\frac{y^{1/2}}{x^{1/2}} = \frac{8}{8}$$
$$u(x_B, y_B) = 50$$

- First, use the $MRS = MRT$ equation to solve for y:

$$\frac{y^{1/2}}{x^{1/2}} = 1$$
$$y = x$$

Computing the IE and SE

$$\frac{y^{1/2}}{x^{1/2}} = 1$$
$$y = x$$

- Then, plug this into the utility constraint:

$$x^{1/2}y^{1/2} = 50$$
$$y^{1/2}y^{1/2} = 50$$
$$y = 50$$

- The bundle we're looking for is thus $(x^*, y^*) = (50, 50)$ (Point B)

Computing the IE and SE

	A	B	C
Condition 1:	$MRS = MRT_1$	$MRS = MRT_2$	$MRS = MRT_2$
Condition 2:	$400 = 8x + 2y$	$u_A = u_B$	$400 = 8x + 8y$
Bundle:	$(25, 100)$	$(50, 50)$	$(25, 25)$

- Difference between points A and C gives the TE:
  - TE_x: 0
  - TE_y: -75
- Difference between points A and B gives the SE:
  - SE_x: $+25$
  - SE_y: -50
- Difference between points B and C gives the IE:
  - IE_x: -25
  - IE_y: -25

Computing the IE and SE

	A	B	C
Condition 1:	$MRS = MRT_1$	$MRS = MRT_2$	$MRS = MRT_2$
Condition 2:	$I = p_x x + p_y y$	$u_A = u_B$	$I = p_x^* x + p_y^* y$
Bundle:	(x_A, y_A)	(x_B, y_B)	(x_C, y_C)

- More generally, to compute the IE and SE, we'll need to compute the three bundles above:
  - Bundle A: optimal bundle with original prices
  - Bundle B: tangency point between IC_1 and compensated BL
  - Bundle C: optimal bundle with new prices (p_x^*, p_y^*)

Keeping Tabs on the SE & IE

$p_x \downarrow$	x	y
SE		
IE		
TE		

- When we do the IE and SE decomposition, there is a lot going on
- It is useful to keep tabs on everything using a table
- Let's work through an example and fill this table in as we go
- Let's assume for this example that:
  - x and y are normal: $\frac{\partial x}{\partial I} > 0$ and $\frac{\partial y}{\partial I} > 0$
  - x follows the LOD: $\frac{\partial x}{\partial p_x} < 0$
  - x and y are substitutes: $\frac{\partial y}{\partial p_x} > 0$

Keeping Tabs on the SE & IE

$p_x \downarrow$	x	y
SE	$\uparrow$	$\downarrow$
IE		
TE		

- Suppose that p_x decreases
- Substitution effect:
  - Buy more of the relatively cheaper good (x here)
  - Buy less of the relatively more expensive good (y here)
- Note that the substitution effect is always the same:
  - SE is always positive for good which gets relatively cheaper, negative for good which gets relatively more expensive

Keeping Tabs on the SE & IE

$p_x \downarrow$	x	y
SE	$\uparrow$	$\downarrow$
IE	$\uparrow$	$\uparrow$
TE		

- Now for the income effect:
  - Prices go down, I have more purchasing power
  - In response, I buy more normal goods (and less inferior goods)
- For the IE row, we reference the signs of the income derivatives
  - $\frac{\partial x}{\partial I} > 0 \rightarrow$ more x
  - $\frac{\partial y}{\partial I} > 0 \rightarrow$ more y

$p_x \downarrow$	x	y
SE	$\uparrow$	$\downarrow$
IE	$\uparrow$	$\uparrow$
TE	$\uparrow$	$\downarrow$

- Lastly, the total effect
- For the TE row, we reference the signs of the p_x derivatives
- $\frac{\partial x}{\partial p_x} < 0$, so x increases as p_x decreases
- $\frac{\partial y}{\partial p_x} > 0$, so y decreases as its p_x decreases

Keeping Tabs on the SE & IE

$p_x \downarrow$	x	y
SE	$\uparrow$	$\downarrow$
IE	$\uparrow$	$\uparrow$
TE	$\uparrow$	$\downarrow$

- For y, which of the effects (IE or SE) dominated?
  - SE was negative, while IE was positive
  - TE was negative, so SE must have been stronger than the IE
- What about for x?
  - SE, IE, and TE were all positive
  - Can't tell if the IE or SE was stronger without more info

Summary

$p_x \downarrow$	x	y
SE	$\uparrow$	$\downarrow$
IE	$\frac{\partial x}{\partial I}$	$\frac{\partial y}{\partial I}$
TE	$\frac{\partial x}{\partial p_x}$	$\frac{\partial y}{\partial p_x}$

- In summary, to fill out this table, we need the signs of four derivatives
- Income derivatives give us the IE row
- p_x derivatives give us the TE row

Going Backwards

$p_x \downarrow$	x	y
SE	$\uparrow$	$\downarrow$
IE	$\frac{\partial x}{\partial I}$	$\frac{\partial y}{\partial I}$
TE	$\frac{\partial x}{\partial p_x}$	$\frac{\partial x}{\partial p_y}$

- We just saw that if we have the signs of the derivatives above, we can fill out the table
- We can also do the reverse:
  - If given the signs of IE or TE, we can infer the signs of the corresponding derivatives
- Let's go through an example

Going Backwards (Example)

$p_x \uparrow$	x	y
SE	$\downarrow$	$\uparrow$
IE	$\downarrow$	$\uparrow$
TE	$\downarrow$	$\downarrow$

- Suppose that we've given the table above
- Four questions we can answer:
  1. Is x normal/inferior/income-neutral?
  2. Is y normal/inferior/income-neutral?
  3. Does x follow the LOD or not?
  4. Are x and y comps/subs/unrelated?

Going Backwards (Example)

$p_x \uparrow$	x	y
SE	$\downarrow$	$\uparrow$
IE	$\downarrow$	$\uparrow$
TE	$\downarrow$	$\downarrow$

- Starting with question 1: Is x normal/inferior/income-neutral?
- Here, p_x increased, and x's income effect was negative
- Prices increase $\rightarrow$ buy less normal goods and more inferior goods
- x is a normal good ($\frac{\partial x}{\partial I} > 0$)

Going Backwards (Example)

$p_x \uparrow$	x	y
SE	$\downarrow$	$\uparrow$
IE	$\downarrow$	$\uparrow$
TE	$\downarrow$	$\downarrow$

- Question 2: Is y normal/inferior/income-neutral?
- Here, p_x increased, and y's income effect was positive
- Prices increase $\rightarrow$ but less normal goods and more inferior goods
- y is an inferior good ($\frac{\partial y}{\partial I} < 0$)

Going Backwards (Example)

$p_x \uparrow$	x	y
SE	$\downarrow$	$\uparrow$
IE	$\downarrow$	$\uparrow$
TE	$\downarrow$	$\downarrow$

- Question 3: Does x follow the law of demand?
- p_x increased, and x's total effect was negative
- x decreased in response to an increase in its price, so x follows the law of demand ($\frac{\partial x}{\partial p_x} < 0$)

Going Backwards (Example)

$p_x \uparrow$	x	y
SE	$\downarrow$	$\uparrow$
IE	$\downarrow$	$\uparrow$
TE	$\downarrow$	$\downarrow$

- Question 4: Are x and y complements, substitutes, or unrelated?
- p_x increased, and y's total effect was negative
- y decreased in response to an increase in p_x, so y and x are complements ($\frac{\partial y}{\partial p_x} < 0$)