Budget Constraints

ECON 410

May 19, 2023

Introduction

- Economic agents make decisions based upon their **preferences** and whatever **constraints** they face
  - Individuals like consuming goods, but have a limited amount of income
  - Students want to study for their classes, but have a limited amount of time
- In the previous section, we discussed modeling preferences
- Now, we'll talk about modeling constraints
- The constraints we'll consider typically take the form of a *budget constraint*

Budget Constraints

- We'll denote our level of income by I
- p_x and p_y respectively denote the price of x and y
- Given income and prices p_x and p_y, the standard budget constraint is:

$$I \geq p_x x + p_y y$$

- The above inequality is called a *budget constraint*
  - RHS: expenditures
  - LHS: income
- In words: we cannot spend more than we have

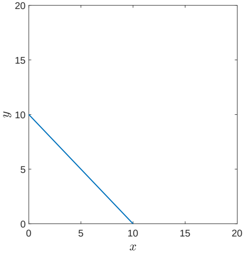
Non-Satiation

- When maximizing $u(x, y)$, we will always focus the case where x and y are good goods
  - Preferences are *monotone*
- Since $MU_x > 0$ and $MU_y > 0$, more is always better
- Intuitively, if more is always better, we should consume as much as x and y possible
- An implication is that we will always spend all of our income:

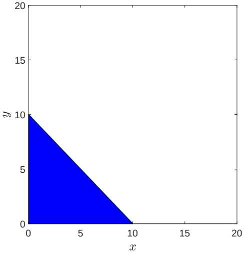
$$I = p_x x + p_y y$$

- Because of this, it is fine in this class to write the budget constraint as an equality rather than an inequality

Budget Constraints Illustrated



(a) Budget Line



(b) Budget Set

- The shaded blue area is the *budget set*
  - Set of (x, y) bundles we can afford
- The boundary of the budget set is the *budget line*
  - Set of (x, y) bundles which exhaust all income
- Typically, budget lines are linear (straight lines)

Slope of the Budget Line

- The budget line is given by:

$$I = p_x x + p_y y$$

- Solving for y gives us:

$$y = \frac{I - p_x x}{p_y}$$

- The slope of the budget line is then:

$$\frac{dy}{dx} = -\frac{p_x}{p_y} = -MRT$$

- We refer to the price ratio $\frac{p_x}{p_y}$ as the *marginal rate of transformation* (MRT)

Marginal Rate of Transformation

$$MRT = \frac{p_x}{p_y}$$

- The MRT summarizes the relative cost of x and y
- If x is very expensive relative to y, then MRT is high
- If x is very cheap relative to y, then MRT is low

Marginal Rate of Transformation

- As p_x, increases, the MRT increases:

$$\frac{\partial MRT}{\partial p_x} = \frac{1}{p_y} > 0$$

- As p_y increases, the MRT decreases:

$$\frac{\partial MRT}{\partial p_x} = -\frac{p_x}{p_y^2} < 0$$

- As income I increases, the MRT is unchanged:

$$\frac{\partial MRT}{\partial I} = 0$$

Intercepts of the Budget Line

$$I = p_x x + p_y y$$

- Setting $y = 0$ and solving for x gives us the x intercept:

$$x = \frac{I}{p_x} \quad \text{when } y = 0$$

- If we spend all our income on x, we could buy $\frac{I}{p_x}$ units
- Similarly, setting $x = 0$ and solving for y gives us the y intercept:

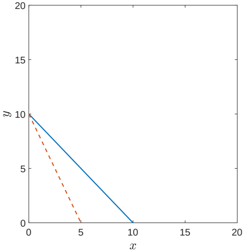
$$y = \frac{I}{p_y} \quad \text{when } x = 0$$

- If we spend all our income on y, we could buy $\frac{I}{p_y}$ units

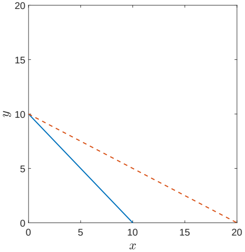
Intercepts of the Budget Line

- The x intercept is given by $\frac{I}{p_x}$
- Notice that as:
  - p_x increases, x intercept decreases
  - As I increases, x intercept increases
  - As p_y increases, x intercept is unchanged
- The y intercept is given by $\frac{I}{p_y}$
- Notice that as:
  - p_y increases, y intercept decreases
  - As I increases, y intercept increases
  - As p_x increases, y intercept is unchanged

Budget Lines - Comparative Statics



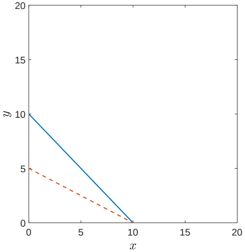
(c) $p_x \uparrow$



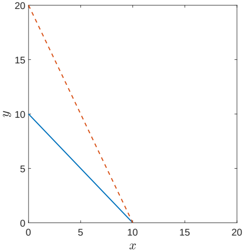
(d) $p_x \downarrow$

- If p_x increases, all else equal, the BL rotates inwards
- If p_x decreases, all else equal, the BL rotates outwards
- No change in y intercept

Budget Lines - Comparative Statics



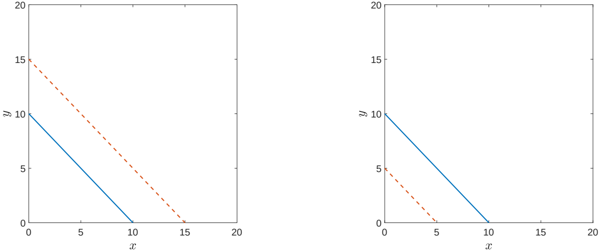
(e) $p_y \uparrow$



(f) $p_y \downarrow$

- If p_y increases, all else equal, the BL rotates inwards
- If p_y decreases, all else equal, the BL rotates outwards
- No change in x intercept

Budget Lines - Comparative Statics



(g) $I \uparrow$ (h) $I \downarrow$

- If I increases, all else equal, the BL shifts outwards
- If p_x decreases, all else equal, the BL shifts inwards
- No change in slope

Taxes and Subsidies

- Taxes and subsidies are two common policies which impact agents' purchasing power, and are very easy to build into budget constraints.
- Suppose a tax τ is imposed on good x
  - For each unit of x we purchase, we must pay tax τ
- The budget line in this setting is:

$$I = p_x x + p_y y + \overbrace{\tau x}^{\text{tax}}$$
$$I = (p_x + \tau)x + p_y y$$

- A tax on x effectively raises its price from p_x to $p_x + \tau$

Taxes and Subsidies

- Alternatively, suppose an tax τ is imposed on income I
  - For each unit of income I, we must pay tax τ
- The budget line in this setting is:

$$I - \overbrace{\tau I}^{tax} = p_x x + p_y y$$
$$(1 - \tau)I = p_x x + p_y y$$

- An income tax effectively lowers income from I to $(1 - \tau)I$

Taxes and Subsidies

- Subsidies work the opposite way that a tax does
- Suppose a subsidy s is imposed on good x
  - For each unit of x we purchase, we are reimbursed s
- The budget line in this setting is:

$$I = p_x x + p_y y - \overbrace{sx}^{\text{subsidy}}$$
$$I = (p_x - s)x + p_y y$$

- A tax effectively lowers the price of x from p_x to $p_x - s$

Other Flavors of Constraints

- While we'll focus mostly on budget constraints, many other types of constraints pop up in economics
- For example, in labor economics, people often consider time constraints
- If T is the amount of hours we have in a day, h is the amount of time spent on work, and ℓ is the amount of time spent on leisure, a typical time constraint looks like:

$$T = h + \ell$$

Other Flavors of Constraints

- Perhaps a company is deciding how much of its profit π to retain R, or give out as dividends D:

$$\pi = R + D$$

- Or, a university has S available seats which can be given to in-state students I, or out-of-state students O:

$$S = I + O$$

- The point is, constraints don't necessarily need to involve income and prices. The "correct" form of a constraint depends on the type of problem you want to solve.