Profit Maximization

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Introduction

- When we started this section of the class, we said that we typically model firms as profit maximizers
- In general, firm profits are given by:

$$\pi(q) = TR(p, q) - c(q)$$

- $TR(p, q)$ is total revenue, which depends on output q and its price p
- The form of $TR(p, q)$ may be different depending on market structure
    - We'll talk about this later
- $c(q)$ is the cost function we've seen before

Profit Maximization

$$\pi(q) = TR(p, q) - c(q)$$

- We've talked about how to optimize costs
- Now, let's assume that firms have already optimized their costs, and are moving on to decide how much to produce
  - $c(q)$ tells firms the minimum cost of producing q
  - Given this, how much q should they produce?
- They select the level of q which maximizes their profit

Profit Maximization

$$\pi(q) = TR(p,q) - c(q)$$

- Remember, to maximize something, we take its first derivative (wrt what we're choosing), and set it equal to zero
- The derivative of the first term (wrt q) is: $\frac{\partial TR}{\partial q} = MR(p,q)$
- $MR(p,q)$, marginal revenue, gives the additional revenue generated from producing one more q
- The derivative of the second term (wrt q) is: $\frac{\partial c}{\partial q} = MC(q)$
- As we've seen, $MC(q)$ is the marginal cost, which gives the cost of producing one additional unit of q

$MR = MC$

- The optimality condition of the profit maximization problem is:

$$MR(p, q) = MC(q)$$

- Firms produce until marginal costs equal marginal revenue
- The quantity q which solves the equation above defines the firm's *supply curve*
- Intuitively, why is this the optimality condition?

$MR = MC$

- Suppose that $MR(p, q) > MC(q)$. This means that producing one more q would increase revenue more than costs
  - Increasing production would increase profits
- Suppose instead that $MR(p, q) < MC(q)$ Then, producing one more q would increase costs more than revenue.
  - Decreasing production would increase profits
- The only time when there is no better level of q to choose is when q satisfies $MR(p, q) = MC(q)$

Profit Maximization (Example)

- Let's suppose for example that a firm faces the cost function:

$$c(q) = 2 + q^2$$

- Assume that total revenue is given simply by price p time quantity q:

$$TR(p, q) = pq$$

- Firm profits are then given by:

$$\begin{aligned} \pi &= pq - c(q) \\ &= pq - 2 - q^2 \end{aligned}$$

Profit Maximization (Example)

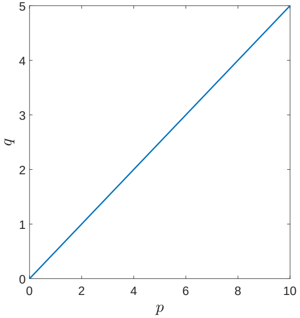
$$\pi = pq - 2 - q^2$$

- We want to maximize with respect to q. Setting $\frac{\partial \pi}{\partial q} = 0$:

$$p - 2q = 0$$
$$q^* = \frac{p}{2}$$

- The profit-maximizing quantity is thus given by $q^* = \frac{p}{2}$
- Note that $MR = p$ and $MC = 2q$ in this example
  - The two are always equal at the optimum

Supply Curve



- The supply curve $q = f(p)$ describes how quantity (i.e. supply) changes with prices
- The supply curve in this example was $q = \frac{p}{2}$
- Generally, supply curves slope up: $\frac{\partial q}{\partial p} > 0$
  - "Law of supply"

Shutdown Decision

- Conditional on producing anything, the optimal level of production q^* satisfies:

$$MR(p, q^*) = MC(q^*)$$

- Notice the *conditional on producing anything* qualifier
- When is it optimal to produce something as opposed to nothing?
- This is the question underlying firms' *shutdown decisions*
- Two options when making the shutdown decision:
  - Produce the q^* satisfying $MR(p, q^*) = MC(q^*)$
  - Shut down and produce nothing

Shutdown Decision

$$c(q) = FC + VC(q)$$

- Costs are generally of the form above
- No matter how much q firms produce, even $q = 0$, they always incur FC
- Variable costs are only incurred if $q > 0$
- When is it optimal to produce something as opposed to nothing?

Shutdown Decision

- If firms produce nothing, $q = 0$, total profit is given by:

$$\pi(0) = TR(p, 0) - c(0) = -FC$$

- If producing q^*, total profit is given by:

$$\pi(q^*) = TR(p, q^*) - c(q^*) = TR(p, q^*) - FC - VC(q^*)$$

- Producing q^* is better than producing $q = 0$ if:

$$\pi(q^*) > \pi(0)$$
$$TR(p, q^*) > VC(q^*)$$

Shutdown Decision

$$TR(p, q^*) > VC(q^*)$$

- If total revenue covers variable costs, it is optimal to stay open and continue producing q^*
- Otherwise, firm should shut down
- The shutdown condition is thus:

$$TR(p, q^*) < VC(q^*)$$

Shutdown Decision

- It is optimal for the firm to shut down if:

$$TR(p, q^*) < VC(q^*)$$

- Notice that the condition does **not** require $TR(p, q^*) < c(q^*)$
- In other words, even if total revenue is less than total costs, it may be optimal to remain open and incur a loss
- Why would firms stay open if they are incurring losses?
- **Intuition:** If a firm is making more than their variable costs, they can use the extra money to gradually pay down their fixed costs. Eventually (in the long run) they'll make a profit.

Taking Inventory

- Firm profits are given by:

$$\pi(q) = TR(p, q) - c(q)$$

- The optimal production level q^* satisfies:

$$MR(p, q^*) = MC(q^*)$$

- As long as $TR(p, q^*) > VC(q^*)$, the best the firm can do is continue operations and produce q^*

- Otherwise, the firm should shut down (i.e produce $q = 0$)

Do firms really only care about profits?

- We've said that we typically model firms as profit maximizers
  - In this case, potential profits are the only thing impacting firms' decisions
- However, it may be reasonable to imagine firms in the real world caring about more than just profits
- This is no problem to handle. We can simply add pieces to firms' objective

Example: Pollution

- Suppose that a firm cares deeply about the environment, but recognizes their production process generates some pollution
- Their preferences may be described by:

$$y(q) = \pi(q) - d(q)$$

- Where $d(q)$ describes the amount of pollution conditional on producing q units of output
- How does their distaste for pollution impact their production decision?

Example: Pollution

$$y(q) = \pi(q) - d(q) = TR(p, q) - c(q) - d(q)$$

- Maximizing with respect to q:

$$\begin{aligned} MR(p, q) - c'(q) - d'(q) &= 0 \\ MR(p, q) &= c'(q) + d'(q) \\ MR(p, q) &= MC(q) \end{aligned}$$

- The marginal cost of production now has two components:
  - Explicit marginal cost $c'(q)$
  - Marginal pollution $d'(q)$
- Distaste for pollution is essentially absorbed into firms' marginal costs.
  - Optimality condition does not change

Taxes & Subsidies

- Just like we saw when working with budget constraints, taxes & subsidies are very easy to model
- Let's go through an example
- Suppose the firm faces the following cost function:

$$c(q) = 2 + q^2$$

- Total profits are given by:

$$\pi(q) = pq - c(q) = pq - 2 - q^2$$

- Optimal quantity is $q^* = \frac{p}{2}$

Taxes & Subsidies

- Suppose now that a tax $\tau > 0$ is imposed on each unit of output q
- This is simply absorbed into the cost function:

$$c(q) = 2 + q^2 + \tau q$$

- Firm profits are now given by:

$$\pi(q) = pq - c(q) = pq - 2 - q^2 - \tau q$$

- Optimal quantity is now given by: $q^* = \frac{p-\tau}{2}$ (less than before)
- Taxes on production decrease supply

Taxes & Subsidies

- Suppose alternatively that a subsidy $s > 0$ is imposed on each unit of output q
- Firm's cost function is now given by:

$$c(q) = 2 + q^2 - sq$$

- Firm profits are given by:

$$\pi(q) = pq - c(q) = pq - 2 - q^2 + sq$$

- Optimal quantity is now given by: $q^* = \frac{p+s}{2}$ (more than before)
- Production subsidies increase supply

Taxes & Subsidies

- Suppose instead that a tax $\tau > 0$ is imposed on each unit of revenue the firm makes
- Total profits are now given by:

$$\begin{aligned}\pi(q) &= pq - 2 - q^2 - \tau pq \\ &= (1-\tau)pq - 2 - q^2\end{aligned}$$

- Optimal quantity is given by: $q^* = \frac{(1-\tau)p}{2}$ (less than before)
- Revenue taxes decrease supply
  - Not as lucrative to produce q, so firms produce less

Taxes & Subsidies

- Finally, suppose a tax $\tau > 0$ is imposed on each unit of profit
- $\pi(q)$ is now given by:

$$\pi(q) = (1 - \tau)(pq - 2 - q^2)$$

- Optimal quantity is given by: $q^* = \frac{p}{2}$ (same as before)
- Taxes on profits have no impact on supply

Aggregation

- Markets typically feature many firms
- Often, we are interested in "aggregate" supply
  - Supply within an entire market, not just one firm's supply
- Suppose a market has N firms
- Each firm has supply function:

$$q_i(p) \quad \text{for} \quad i = 1, \ldots, N$$

- Aggregate supply $q(p)$ is obtained by simply adding up the individual supplies:

$$q(p) = \sum_{i=1}^{N} q_i(p) = q_1(p) + q_2(p) + \ldots + q_N(p)$$

Aggregation

- For example, consider a market firm two firms: Firm 1 & Firm 2
- Each has the supply function:

$$q_1 = 2 + \frac{p}{2}$$
$$q_2 = 5 + \frac{p}{2}$$

- Aggregate supply q is given by:

$$q = q_1 + q_2 = 7 + p$$